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THE FLOW ABOUT AND BEHIND THREE DIMENSIONAL LIFTING SURFACES AND THE AERODYNAMIC NOISE PRODUCED IN COMPRESSIBLE SUBSONIC FLOWS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The work is proceeding along two separate paths which it is hoped eventually to amalgamate. The first investigation consists of numerical work on the rolling up of the wing trailing vortex system when it is represented by discrete vortices according to a series of models of increasing complexity starting from the two-dimensional elliptic distribution as originally treated by Westwater and proceeding by stages through various lifting-line and lifting-surface models, including some exhibiting non-linear lift characteristics. The second project is a detailed investigation of the flow field near wing tips and wake edges,

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including thickness effects. It is felt that the precise flow field near wing tips and the wake edges must be understood if the rolling up and structure of the trailing-vortex cores is to be properly studied.

(a) SUMMARY OF THE RESEARCH PROGRAM

1. Wing Tip Vortices Studies

The method for the calculation of lift distribution and the near vortex wake described in the previous progress reports have been generalized for the calculation which includes both low aspect ratio and high aspect ratio wings. Additional computer programs are being developed to account for the vortex structure in the trailing wake. Results for linear lift variation with angle of attack were obtained. Calculations for the nonlinear lift variation for the case of straight trailing vortices have been completed for the case of straight shedded vortices. Good agreement between the calculations and experimental data is obtained.

Calculations using a finite diameter core in the discreet vortices has been programmed and computations of lift distributions and vortex trail are underway.

The calculation of the lift distribution on a circular wing discussed in PR 6 is progressing. Some of the computer programs are already working and results are being obtained. The results are being compared with exact calculations for circular wings.

This program is described in Ref. 3, which is presented as Appendix A to this report.

2. Aerodynamic Noise Studies

A new jet experimental set up is in its final stages of construction. Upon completion of the system, the experimental program of noise measurements will be continued.

3(a) RESEARCH PLAN FOR NEXT PERIOD

1. Calculation of the lift distribution on a circular wing planform using the potential equation developed in Ref. 1.

2. Calculation of the rolling up of a finite thickness vortex layer behind a wing including the nonlinear lift variation.

3. Completion of the construction of the system for measurements of the noise generated by a subsonic swirling jet.

(b) PUBLICATION UNDER PRESENT GRANT

1. H. Portnoy and J. Rom, "The Flow Near the Tip and Wake Edges of a Lifting Wing with Trailing-Edge Separation", T.A.E. Report No. 132, August, 1971.

2. J. Rom and C. Zorea, "The Calculation of the Lift Distribution and the Near Vortex Wake Behind High and Low Aspect Ratio Wings in Subsonic Flow", T.A.E. Report No. 168, December, 1972.

3. J. Rom,, H. Portnoy and C. Zorea, "Investigations into the Formation of Wing-Tip Vortices"to be presented at Aeromech 41, England, Sept. 17-21, 1973.

1.

INVESTIGATIONS INTO THE FORMATION OF WING-TIP VORTICES

by

J. Rom, H. Portnoy and C. Zorea

SUMMARY

A resumé of the programme of theoretical investigations into wing-tip vortex formation which is now in progress in the Department of Aeronautical Engineering at the Technion, Haifa, is given.

The work is proceeding along two separate paths which it is hoped eventually to amalgamate. The first investigation consists of numerical work on the rolling up of the wing trailing vortex system when it is represented by discrete vortices according to a series of models of increasing complexity starting from the two-dimensional elliptic distribution as originally treated by Westwater and proceeding by stages through various lifting-line and lifting-surface models, including some exhibiting non-linear lift characteristics.

The second project is a detailed investigation of the flow field near wing tips and wake edges, including thickness effects. It is felt that the precise flow field near wing tips and the wake edges must be understood if the rolling up and structrue of the trailing-vortex cores is to be properly studied.

This research has been sponsored in part by the Air Force Office of Scientific Research (AFSC), United States Air Force, through the European Office of Aerospace Research, OAR, United States Air Force under Grant AF OSR 71-2145.

INTRODUCTION

The present programme of research is aimed at resolving some of the problems existing at the moment in the calculation of the form of the vortex wake behind a lifting wing and its influence on the wing pressure distribution.

The reasons for the project are primarily connected with the need for more accurate knowledge of the wake, because of the increasing importance of interactions between aircraft in crowded airlanes[(1)] and the need to minimize these interactions by accelerating the dissipation of the trailing vortices (see figure 1). An important secondary consideration is the more accurate calculation of wing characteristics, especially non-linear effects, by use of a more realistic trailing-vortex model than the traditional planar wake.

Part 1 - Calculations of the Wake Shape and the Associated Pressure Distribution Using Discrete Vortices

In this part of the work the aim has been to build up a series of computer programmes starting from simplified models of the vortex system which have been dealt with by previous authors[(2)-(5)] and proceeding gradually to more realistic representations; at each stage examining the various alternatives and eliminating the practical difficulties, as far as possible, prior to proceeding further.

(a) Two-dimensional Wake Model with Elliptic Spanwise Circulation Distribution

This programme reproduces the early work of Westwater[2] in which the flow in a fixed stationary plane far behind a moving wing with elliptic circulation distribution is identified with the problem of the unsteady motion of a two-dimensional array of point vortices moving under their own mutual influences (see figure 2).

This basic programme enables us to investigate many of the practical problems which occur in all discrete vortex models: whether to use equal strength or equally spaced vortices, the problem of "cross-over" of the vortex trajectories "escape" of the tip vortex and the development of irregular shape of the sheet (see figures 5,6 and 7).

These problems are basically due to the use of a concentrated point vortex model with its associated velocity singularities, plus accumulative effects of rounding-off errors etc. The solutions of these difficulties were found to lie in a proper choice of the number and spacing of the vortices and the size of the time intervals (corresponding to the donwnstream separation of the planes at which the vortex pattern is calculated step by step). The correct combinations were found by a series of numerical experiments. It is important to note that due to the causes mentioned, too many vortices or stations can lead to deterioration of the results rather than convergence on a limiting form.

At a later stage in the work the two-dimensional programme was used to develop the following idea. The actual wake has thickness and is not an infinitesimally thin sheet. A better approximation to the wake than an array of point vortices is, therefore, an array of Rankine vortices. If we assume, therefore, a core diameter fixed by the conditions of constant vorticity per unit cross-sectional area of the core (the density being calculated, say, by using the method of Spreiter and Sacks[(5)] to estimate the ultimate diameters of the final pair of cores), we can impose the condition that, as soon as two cores touch or intersect, the two vortices are replaced by a single one at the centre of gravity of the pair, with core diameter fixed by the same constant density condition. This procedure overcomes the difficulties caused by too close approach of the vortex centres, i.e. "intersection" and "escape", it limits the number of vortices automatically, thus helping to eliminate other irregularities, it preserves the distance of the centre of gravity of each half of the array from the wing plane of symmetry (which is required by Betz's theorem[(6)] for strictly two-dimensional motions but applies fairly well to the three-dimensional wakes examined later, also) and it finally ensures that, when all the vortices are amalgamated far downstream, we end up with a pair with the correct diameters as predicted by the ultimate-core-size theory.

(b) Lifting-line Model with Elliptic Spanwise Circulation Distribution

In the next stage of the work a wing with an elliptic circulation distribution was dealt with, employing a single bound vortex in the wing quarter-chord line (fig. 2). The trailing vortices were continued straight from this line to the traling edge and, from the trailing edge backwards, their rolling up was taken, as a first approximation, to be given by the results of section (a). The corrections necessary to this wake shape to conform with the lifting-line vortex pattern were then calculated by a series of iterations, assuming the elliptic spanwise circulation distribution unchanged throughout. The experience gained in stage (a) was utilized to select the number of vortices etc. and at a later stage the finite-core concept was incorporated here too.

A practical point worth mentioning is as follows. The rolling-up calculation was only carried as far as ten semi-spans downstream of the trailing edge. However, a correction for the influence of the portion of the wake downstream of this station was incorporated. At the ninth and tenth stations the centre of gravity of the vortex-wake cross-section (in the half-span domain) was found and a line through these two points was taken to define the position and direction of a single replacement trailing vortex for each half wing, starting from the tenth station extending to infinity and having the appropriate ultimate

strength (i.e. equal to the value of the mid-span circulation). The induced velocity due to this pair of semi-infinite vortices was taken as an appropriate correction for the influence of the portion of wake downstream of the tenth station on the flow upstream of it. This method was employed in all subsequent stages.

A comparison of results form this programme and that in section (a) is shown in figure 8.

(c) Model Employing a Single Sheet Vortex Lattice

At this stage a lifting-surface theory was introduced to enable us to deal with general wing shapes with some accuracy (see fig. 3).

The standard vortex-lattice technique was employed[(7)-(12)] to start with. The wing was divided into cells by a number of equally spaced streamwise lines and by either equally-spaced constant-percentage-chord lines or by lines perpendicular to the stream, as shown in figure 3a. A bound vortex element was placed at the "quarter-chord" of each box and a control point was established at the middle of its "three-quarter-chord" line. The usual trailing vortices were taken to spring from each bound vortex element and to continue straight downstream in the wing plane, and the unknown vortex strengths were found, in the usual way, by evaluating the downwash at each control point, equating it to the value given by the boundary conditions and solving the resultant system of linear equations.

The wing aerodynamic coefficients were also found from the results, to a linear approximation.

A first approximation to the rolling up was now found by taking the actual spanwise distribution of circulation found (which will generally not be elliptic) and doing a two-dimensional calculation of rolling up, as was done in section (a) for the elliptic case. This was now taken as the first approximation to the rolled-up wake shape starting from the trailing edge. The finite-core model has also been incorporated into this calculation, using a vortex density based on Spreiter and Sack's[5] work and utilizing the C_{D_i} calculated from the linear vortex-lattice solution[12]. As in section (b), the first approximation to the wake shape was then improved iteratively using the calculated vortex strengths. During these iterations the vortex strengths were kept constant. The next stage involved keeping the wake shape constant, re-calculating the vortex strengths and the aerodynamic coefficients and adjusting the vortex core sizes to suit. We then returned to recalculation of the sheet shape by iterations, keeping the vortex strengths fixed, and so on. Some wake shapes calculated by this method are shown in figures 18 and 19.

(d) Model Employing a Multi-Layered Vortex Lattice

In order to deal with wings of all types, including those with leading-edge separation, a multi-layered vortex lattice model was now introduced as the starting point in place of the planar version (see figure 4). This employs the assumption utilized by Bollay[13] and Gersten[14] that the trailing vortices do not leave the wing parallel to its mean plane, but that they immediately leave at an angle $\frac{1}{2}\ \alpha$ to the plane and then extend in straight lines to infinity. The system of bound-vortex elements was set up exactly as in the previous section and the vortex strengths and wing properties were found by the process described there. In this case, however, the results have been found to be sensitive to the choice of swept or unswept bound vortices. Unswept vortices have been found to give results closer to these found from experiment. In dealing with the rolling up the first approximation to the wake shape was not found by the earlier two-dimensional method, but the calculated vortex system with straight trailing vortices was allowed to act on itself to produce the first distortion, followed by re-calculation of the vortex strengths using the distorted wake shape and then re-calculation of the wake shape etc. This last process is still, in fact, in its trial stages and the difficulties of intersection and "escape" of the vortices have been encountered very strongly because of the close vertical separation of the trailing vortices from neighboring layers of bound vortices. The finite-core method is to be introduced into this programme to overcome these troubles.

It is hoped that the final version will be able to reproduce automatically an adequate version of the correct vortex configuration for wings of any shape and aspect ratio.

A number of graphical comparisons of the aerodynamic coefficients as predicted by the various methods described here and by previous work, both theoretical and experimental, are shown in figures 9-17 and in figure 20 the vortex centre position is shown for a rectangular wing.

Part 2 - The Flow Near the Tip and Wake Edge of a Lifting Wing with Trailing-Edge Separation

In order to be able to calculate the dissipation behaviour of the trailing vortex cores properly a knowledge of their inner structure is required. The central part of each core consists of vorticity emanating from the flow near the wing tip and the neighbouring wake edge.

However, it is precisely in the region of the tip and near the wake edge that the results of linearised theory are most unsatisfactory because of the singular behaviour of the solution there. Jordan[15] has recently elucidated the nature of the complicated singularity at the tip point whilst at the wake edge we have the familiar singularity due to incompressible flow round a sharp edge.

In this section of the work the wake-edge and tip singularities were removed by taking into account the finite thickness of the wing and the effective (displacement) thickness of the wing boundary layer and wake. The method of matched asymptotic expansions[16] was used to match an inner, two-dimensional solution with the outer linearised result so as to obtain a uniformly valid expression (see figures 21 and 22). This is combined with earlier results doing the same thing for the rounded leading edge singularity[16] to finally obtain a uniformly valid first order result for the wing potential in all parts of the flow field. The final result is shown in Fig. 23.

Work is now in progress on programming this result to obtain the flow field about the wing and the unrolled wake to a uniform first-order approximation. At a later stage it is hoped to combine the results of the two sections to obtain an idea of the effect of wing thickness and boundary layer development on the rolled up wake structure.

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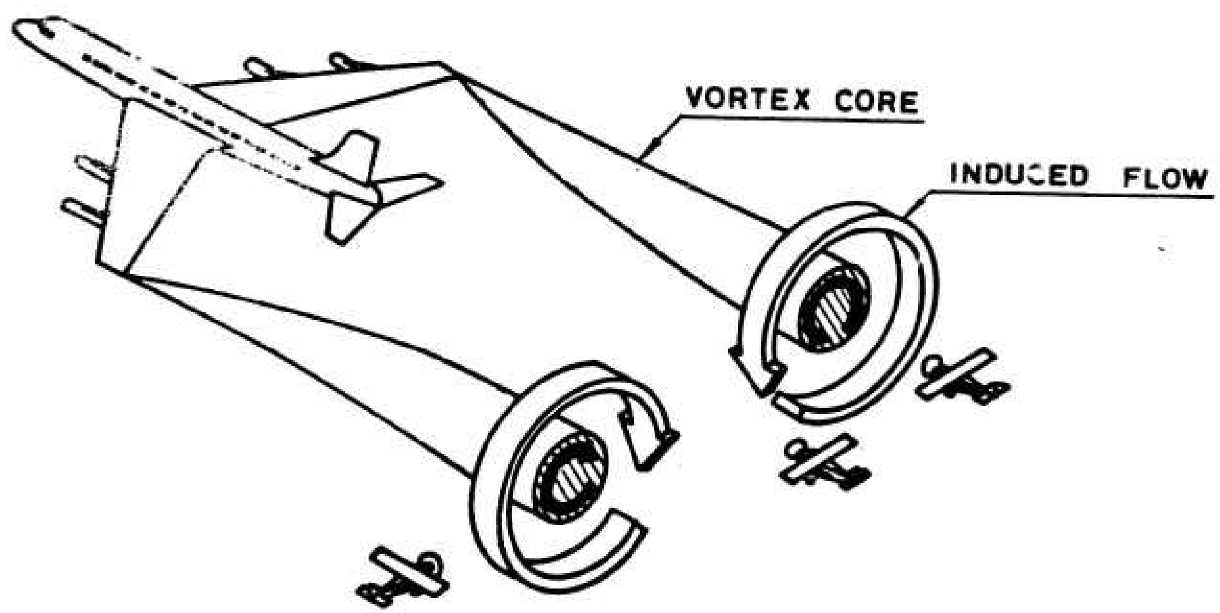


FIGURE 1 - ILLUSTRATION OF TRAILING VORTEX WAKE AND TYPES OF ENCOUNTER (From Ref. 1)

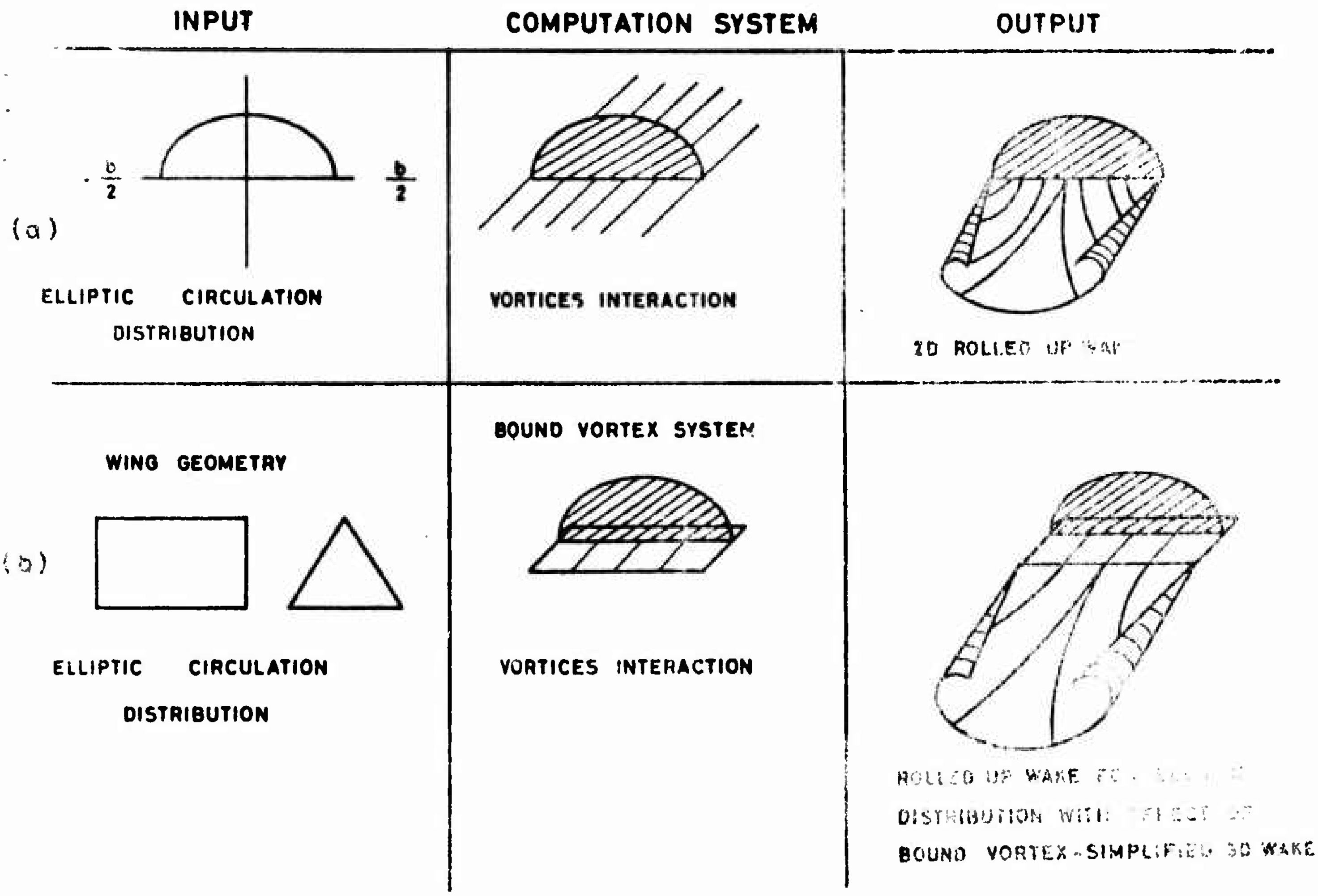


FIG 2 - ELLIPTIC LIFT, VORTEX WAKE CALCULATIONS

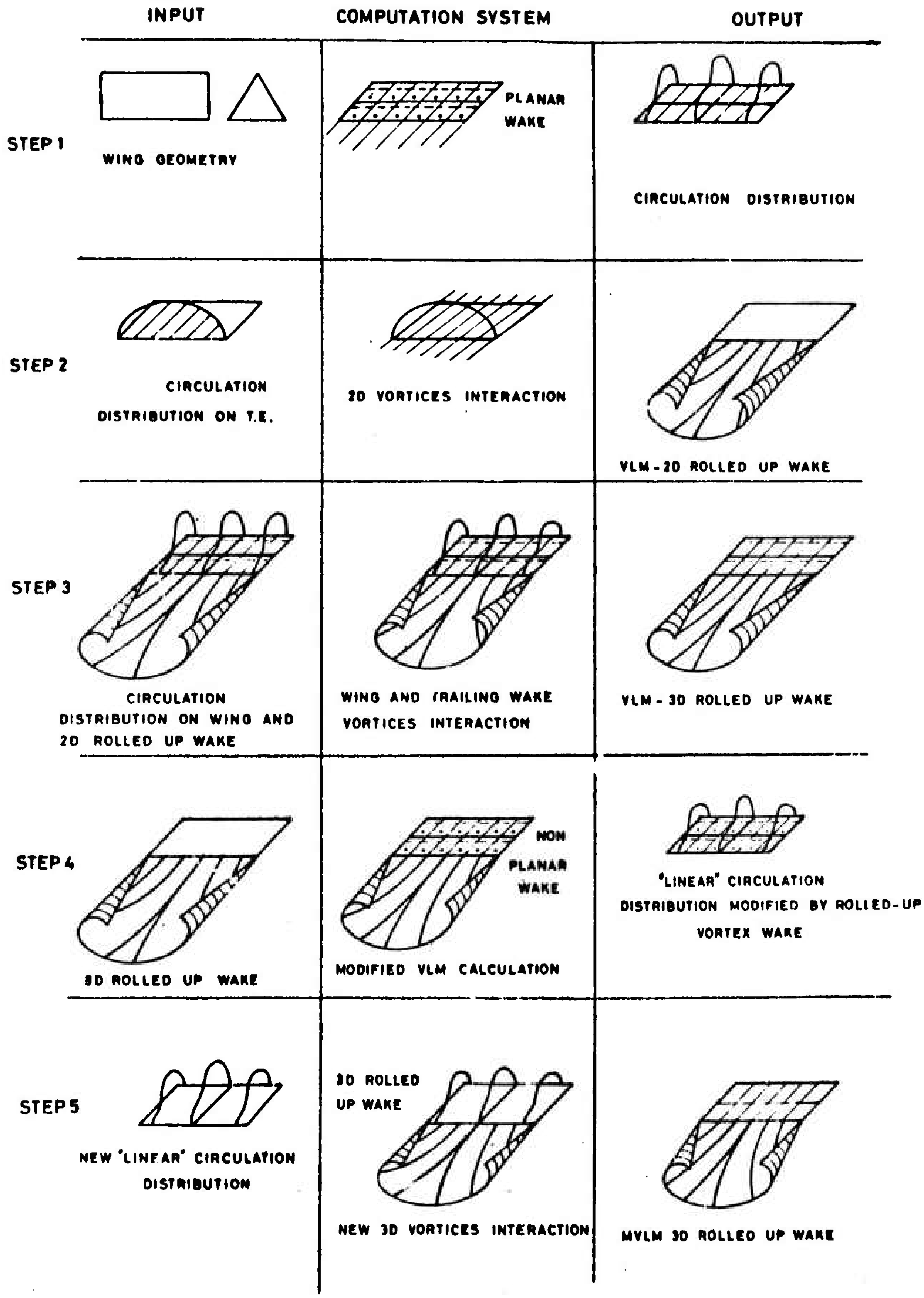
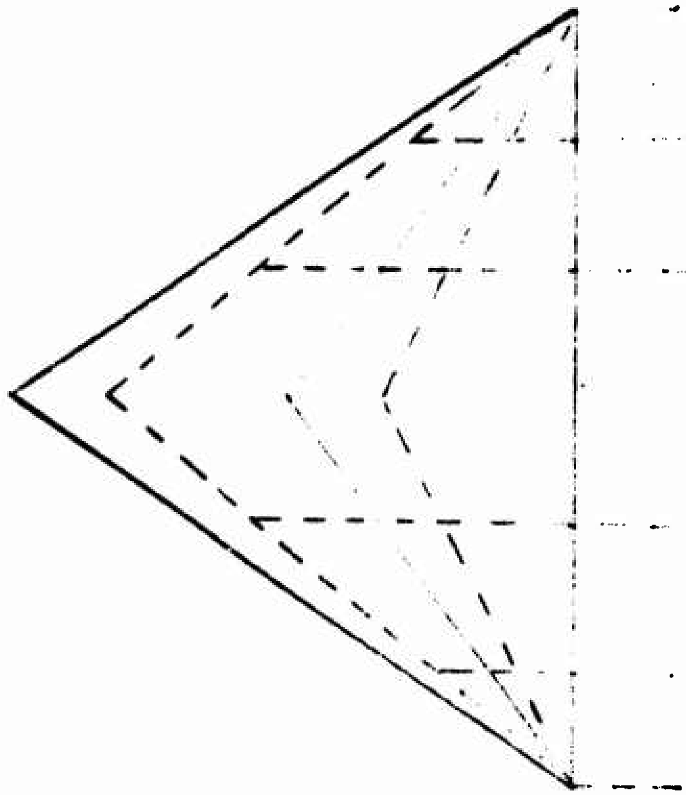
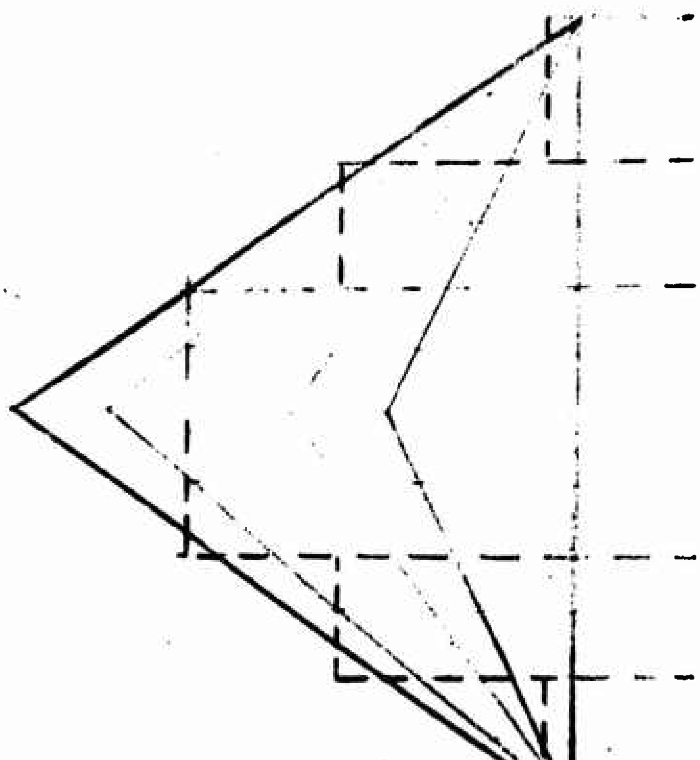


FIG 3 - VLM LIFT VORTEX WAKE CALCULATIONS





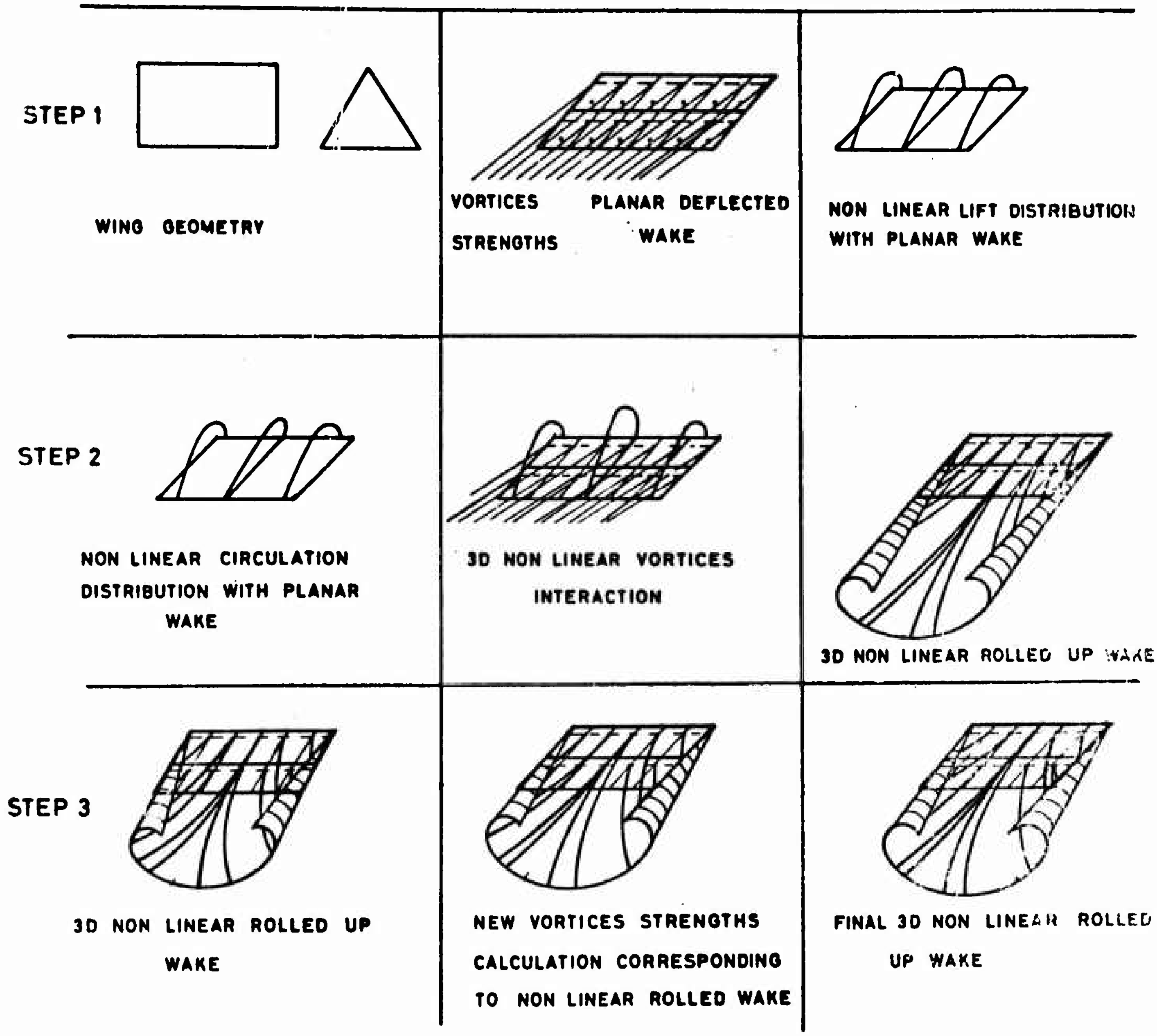


FIG 4 - NON LINEAR VORTEX WAKE CALCULATIONS

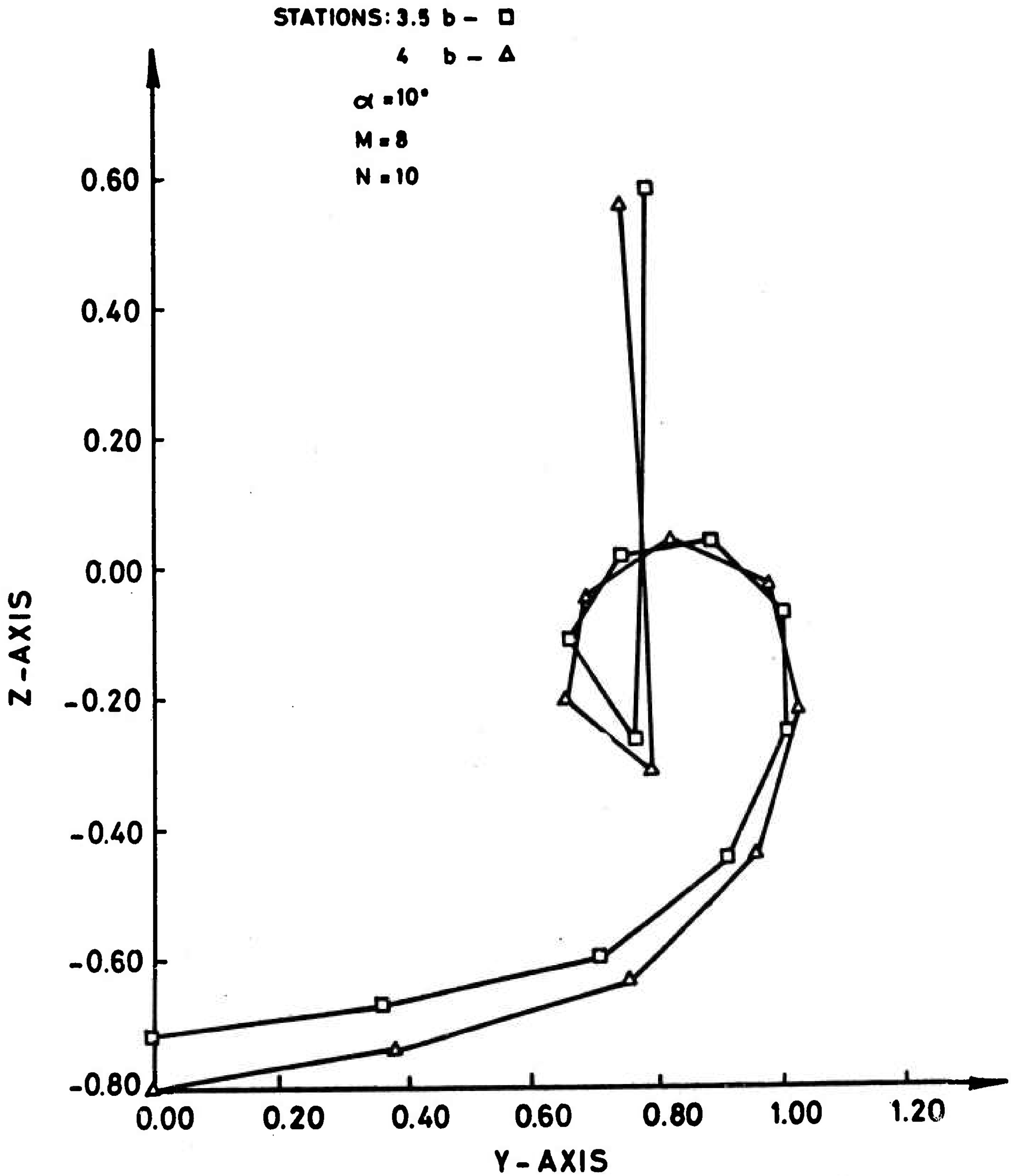


FIGURE 5 - PROBLEMS IN THE CALCULATIONS OF THE VORTEX SHEET ROLL-UP - "ESCAPE" OF THE "TIP" VORTEX.

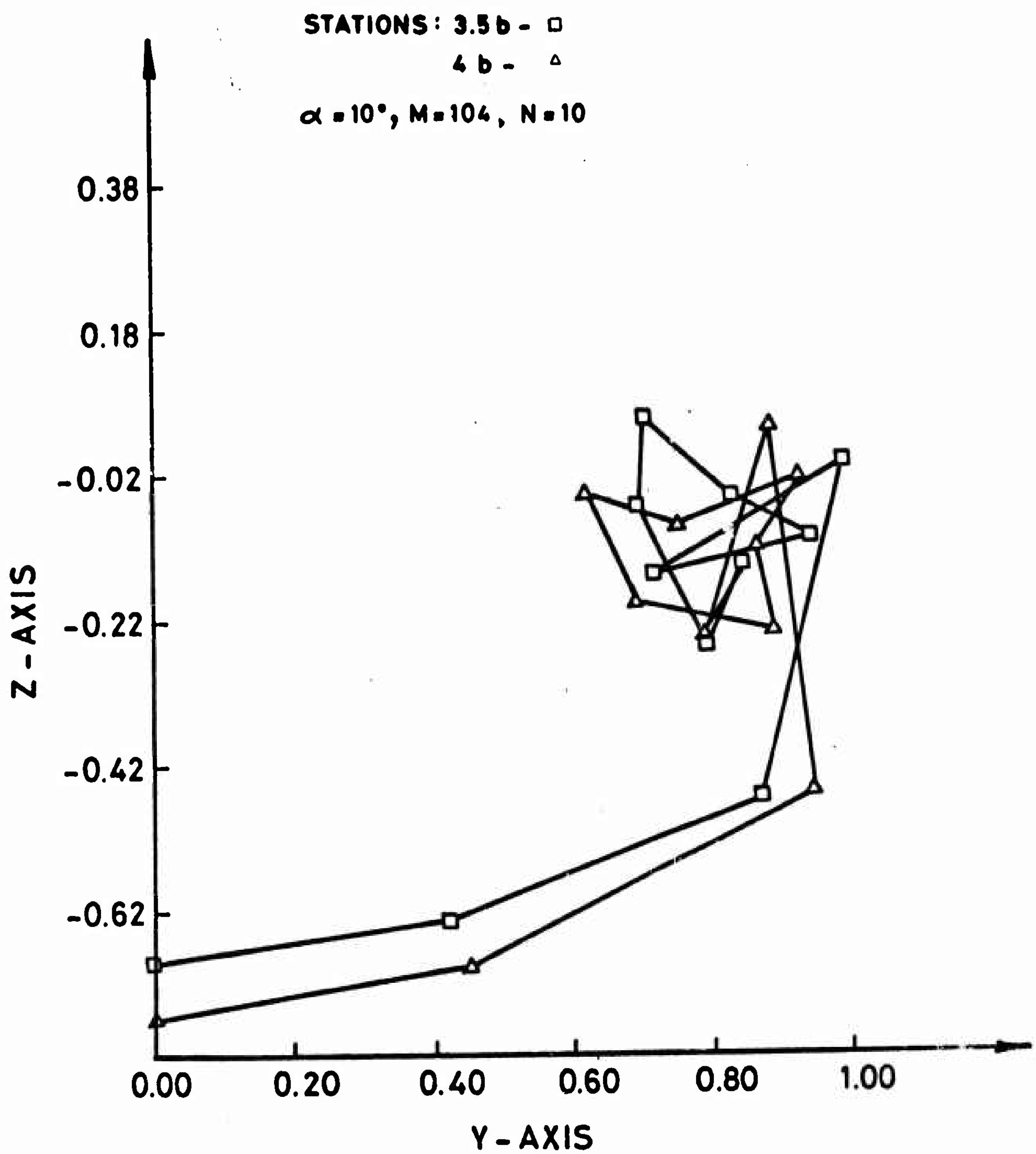


FIGURE 6 - PROBLEMS IN THE CALCULATIONS OF THE VORTEX SHEET ROLL-UP
CROSS-OVER OF VORTEX LINES.

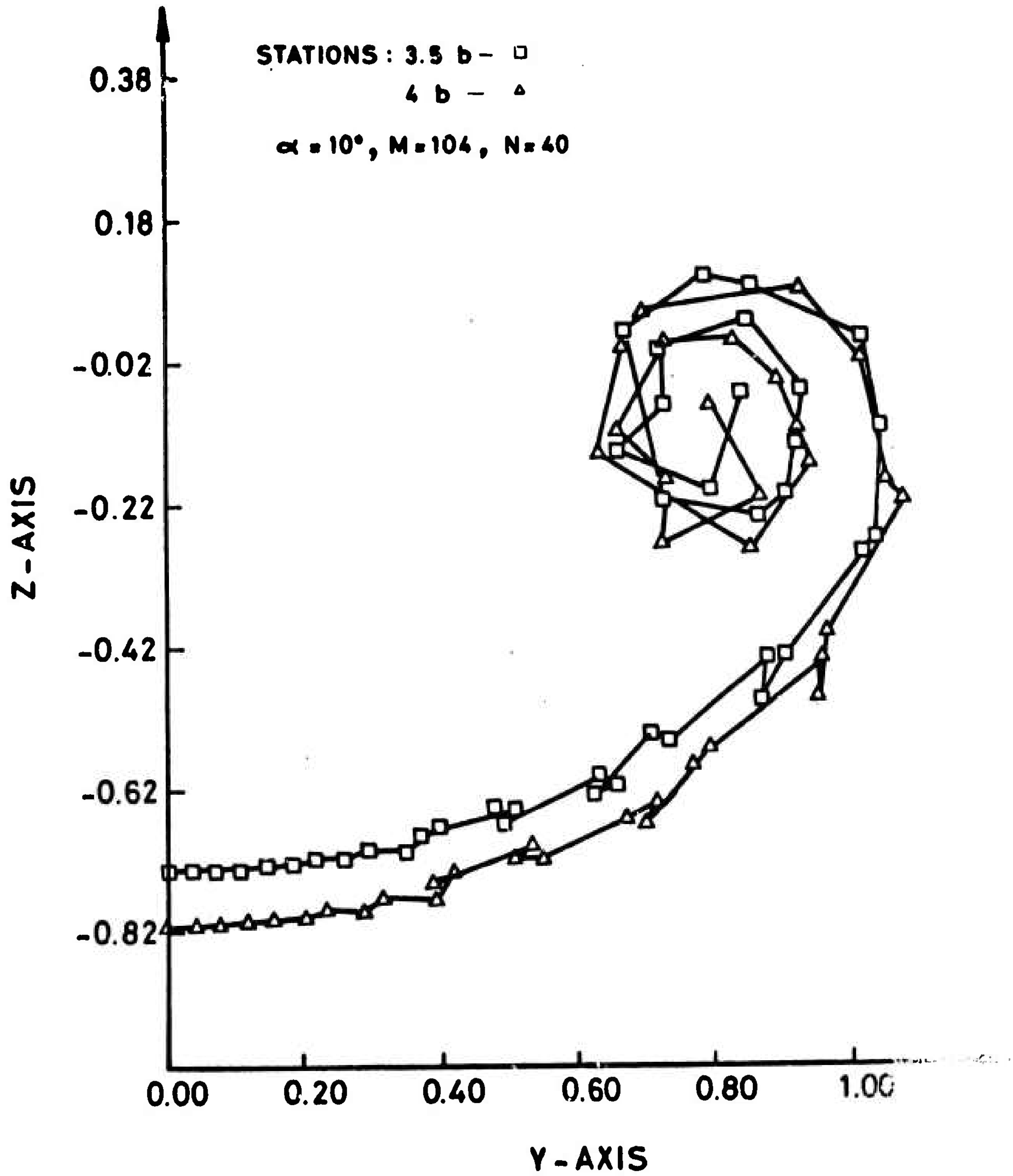


FIGURE 7 - PROBLEMS IN THE CALCULATIONS OF THE VORTEX SHEET ROLL [illegible]
TOO MANY SUBDIVISIONS.

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ELLIPTIC LIFT DISTRIBUTION PROGRAM

EQUAL SPACED VORTICES

Rectangular Wing AR = 3.

$\alpha = 10^{o}$

M = 32

N = 20

□ - 2D

△ - 3D

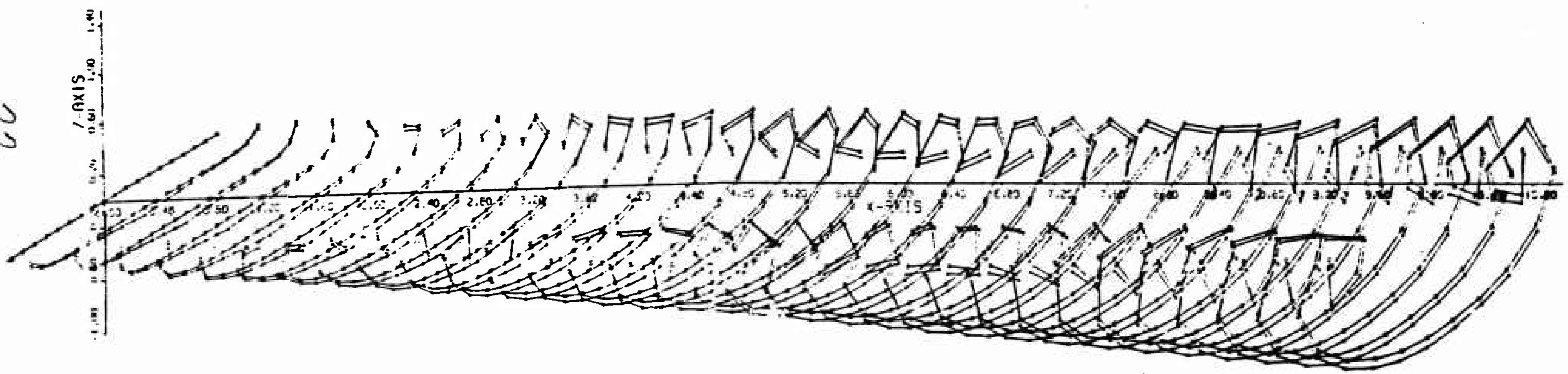

FIGURE 8 - THE TWO AND THREE DIMENSIONAL ROLLED UP WAKE SHAPES CALCULATED BY ELLIPTIC LIFT DISTRIBUTION PROCEDURE. (Figure Axes are given by Half Span Units).

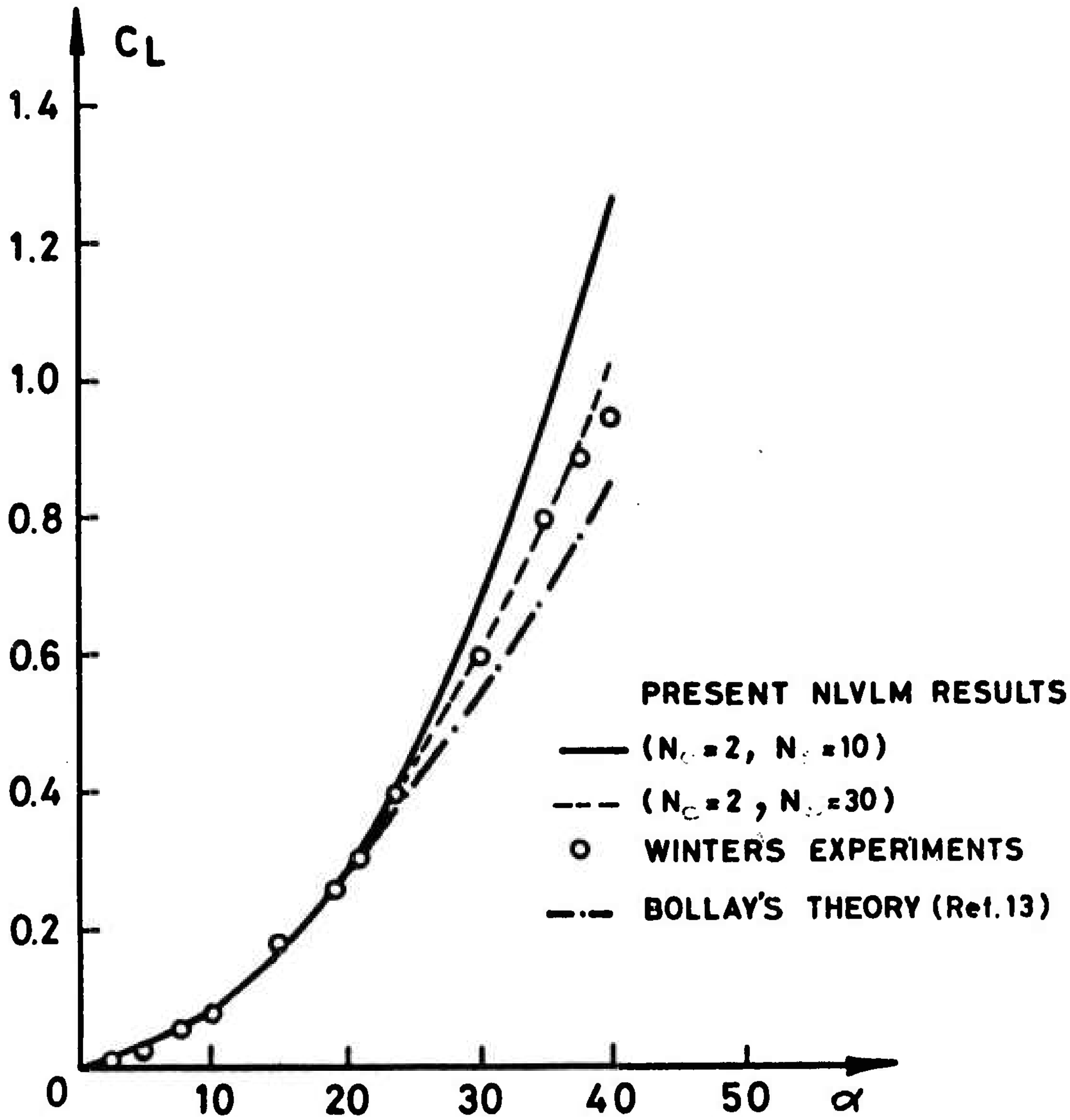

FIGURE 9 - THE LIFT COEFFICIENT CALCULATED BY THE NLVLM PROGRAM COMPARED WITH RESULTS PRESENTED IN REF. 13, FOR RECTANGULAR WING OF AR = 1/30.

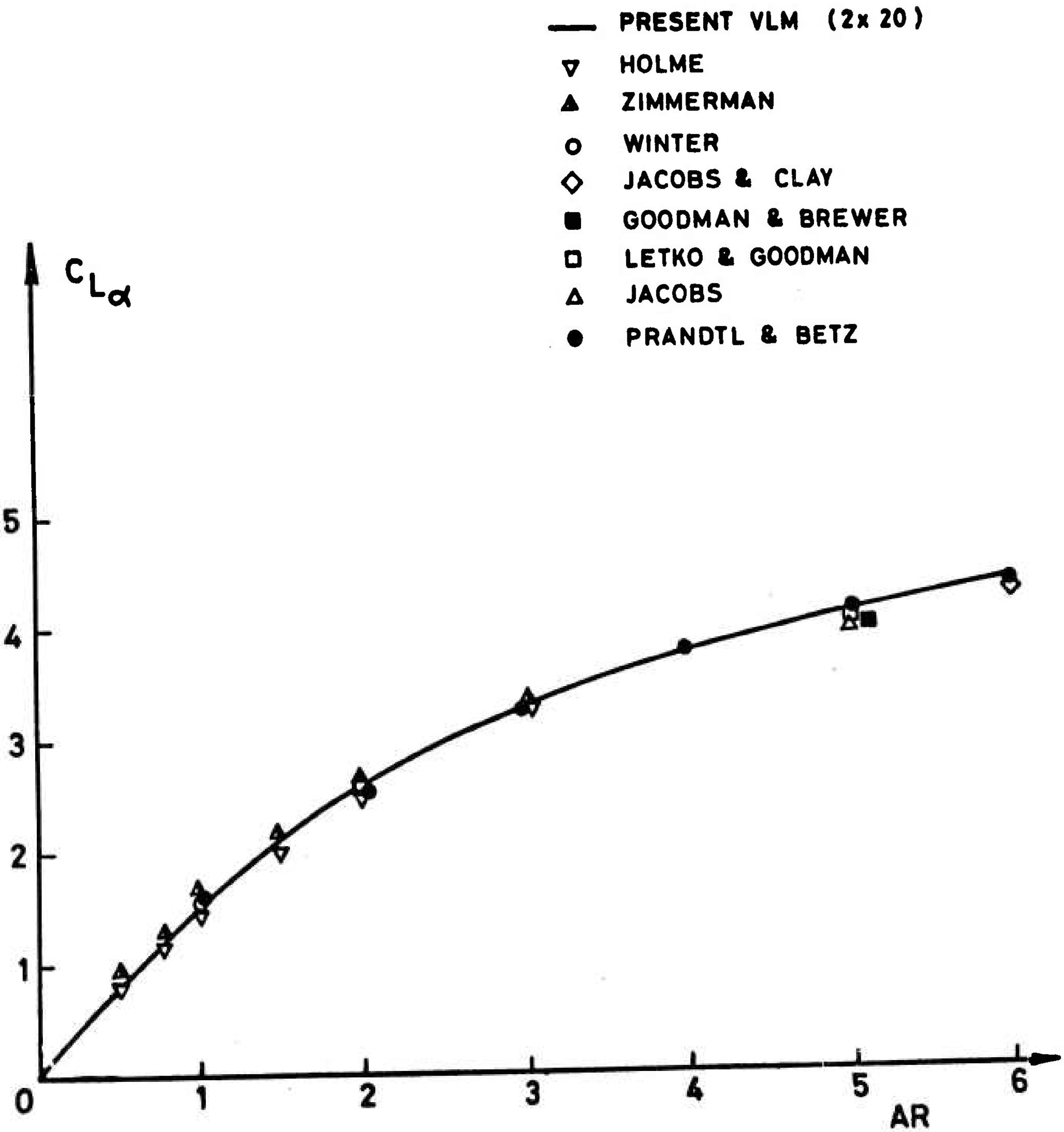

FIGURE 10 - THE LIFT CURVE SLOPE CALCULATED BY THE VLM PROGRAM. COMPARISON WITH EXPERIMENTAL RESULTS (REF. 17, 18) FOR RECTANGULAR WINGS.

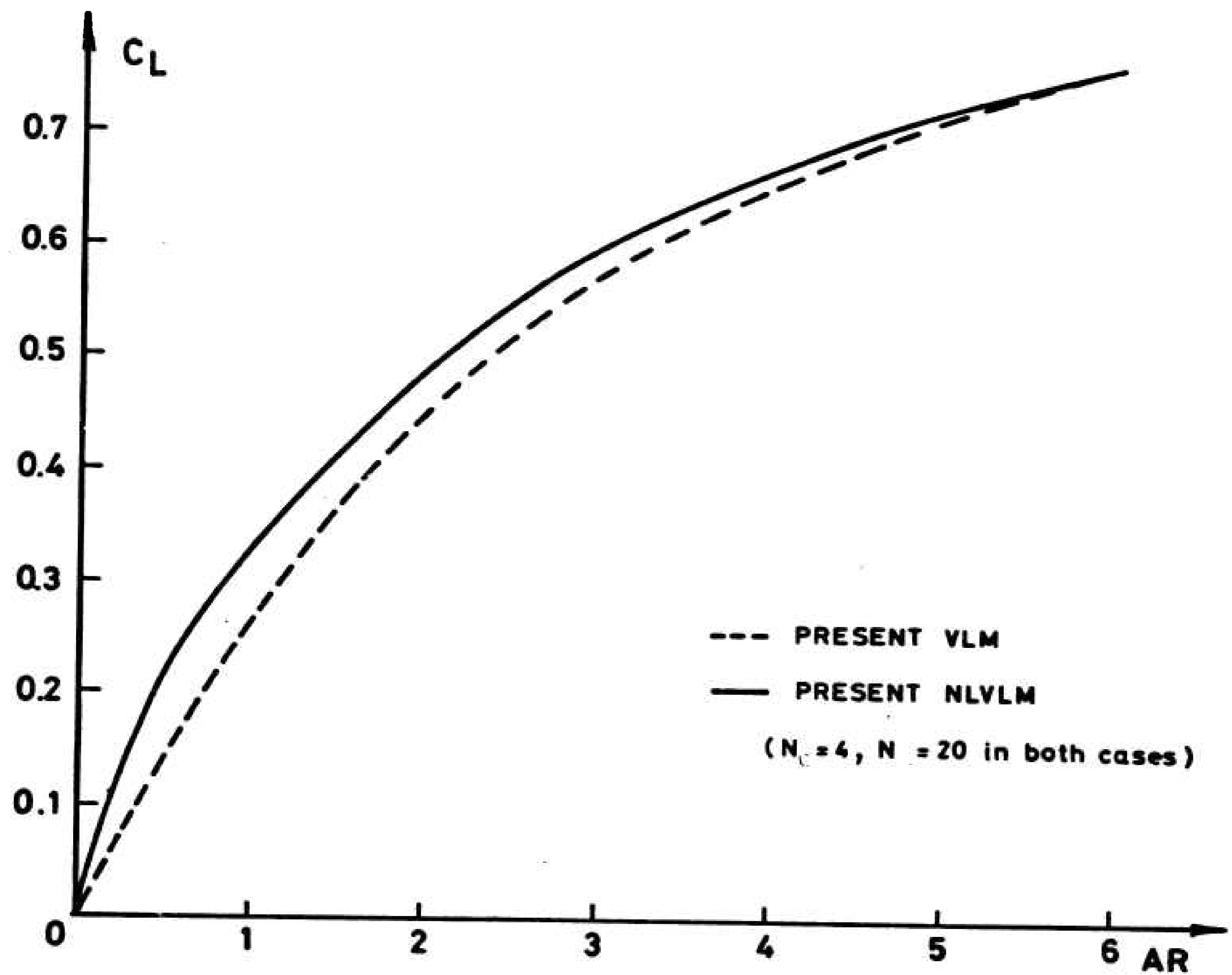

FIGURE 11 - COMPARISON OF THE LINEAR LIFT COEFFICIENT CALCULATED BY THE VLM PROGRAM AND THE NONLINEAR LIFT COEFFICIENT CALCULATED BY THE NLVLM PROGRAM FOR RECTANGULAR WINGS AT [illegible] 10°

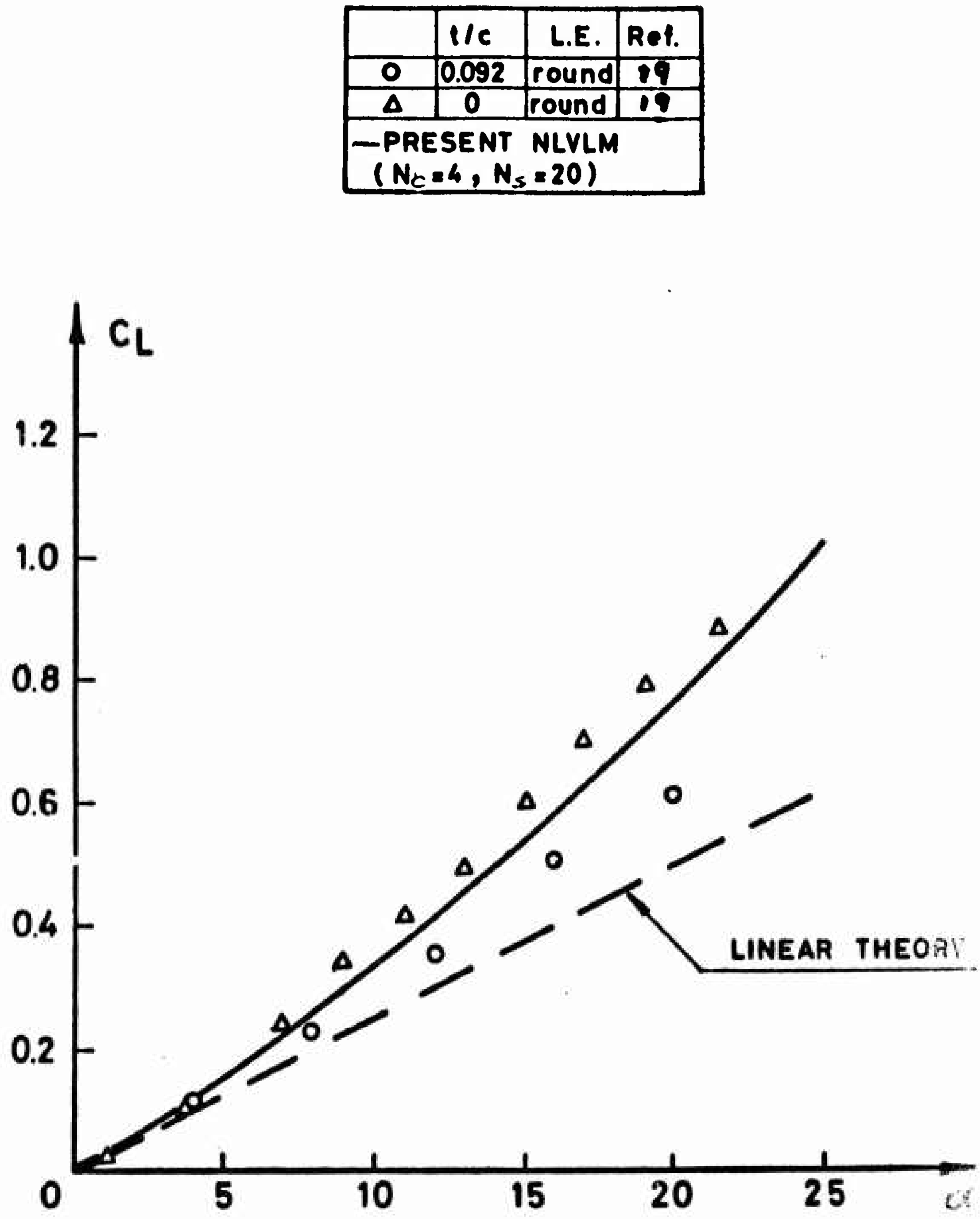

FIGURE 12a - THE LIFT COEFFICIENT OF A RECTANGULAR WING OF AR

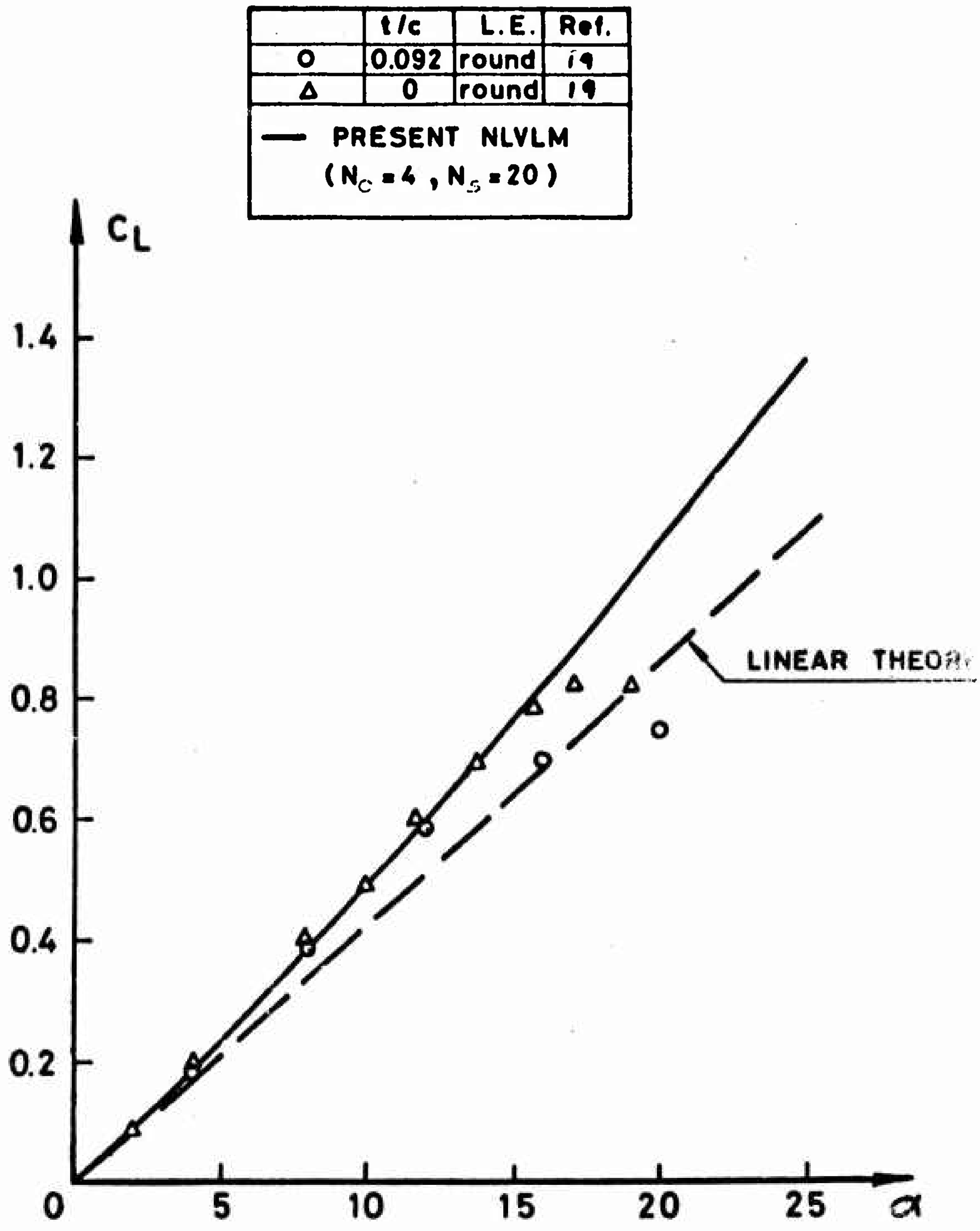

FIGURE 12b -- THE LIFT COEFFICIENT OF A RECTANGULAR WING AR = 2.

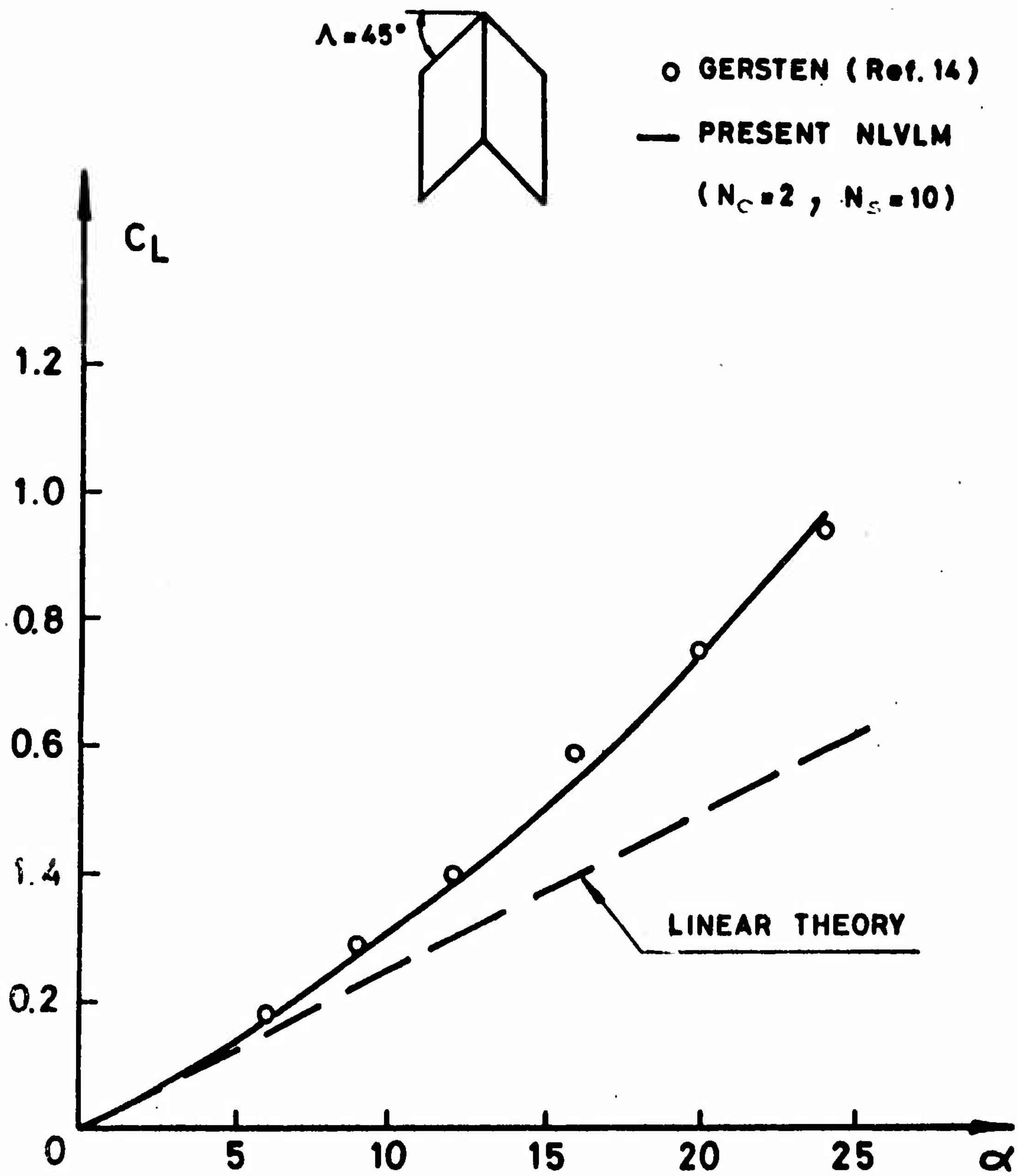

FIGURE 13a - THE LIFT COEFFICIENT OF A SWEPT BACK RECTANGULAR WING OF AR = 1

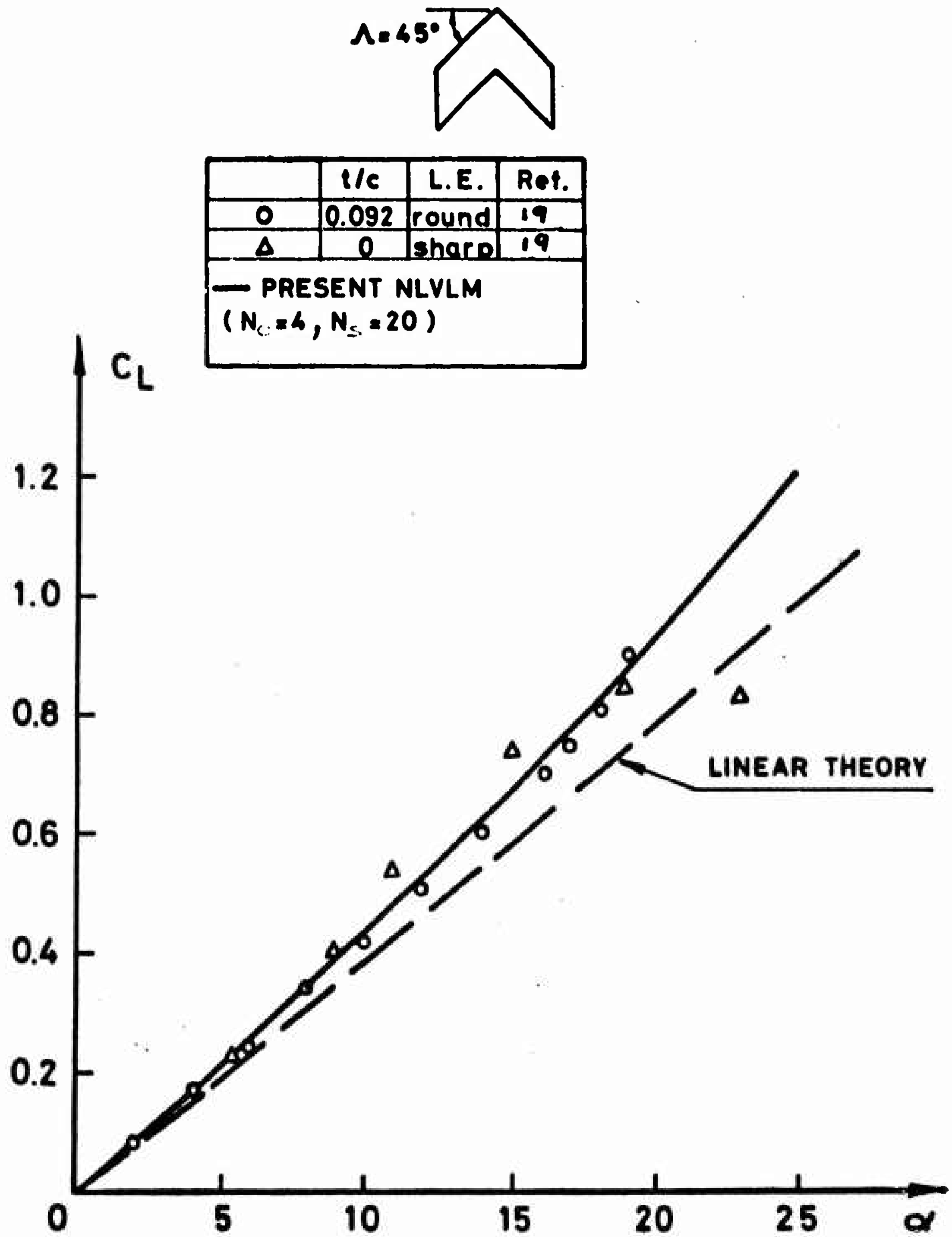

FIGURE 13 b - THE LIFT COEFFICIENT OF SWEPTBACK RECTANGULAR WING OF AR = 2.

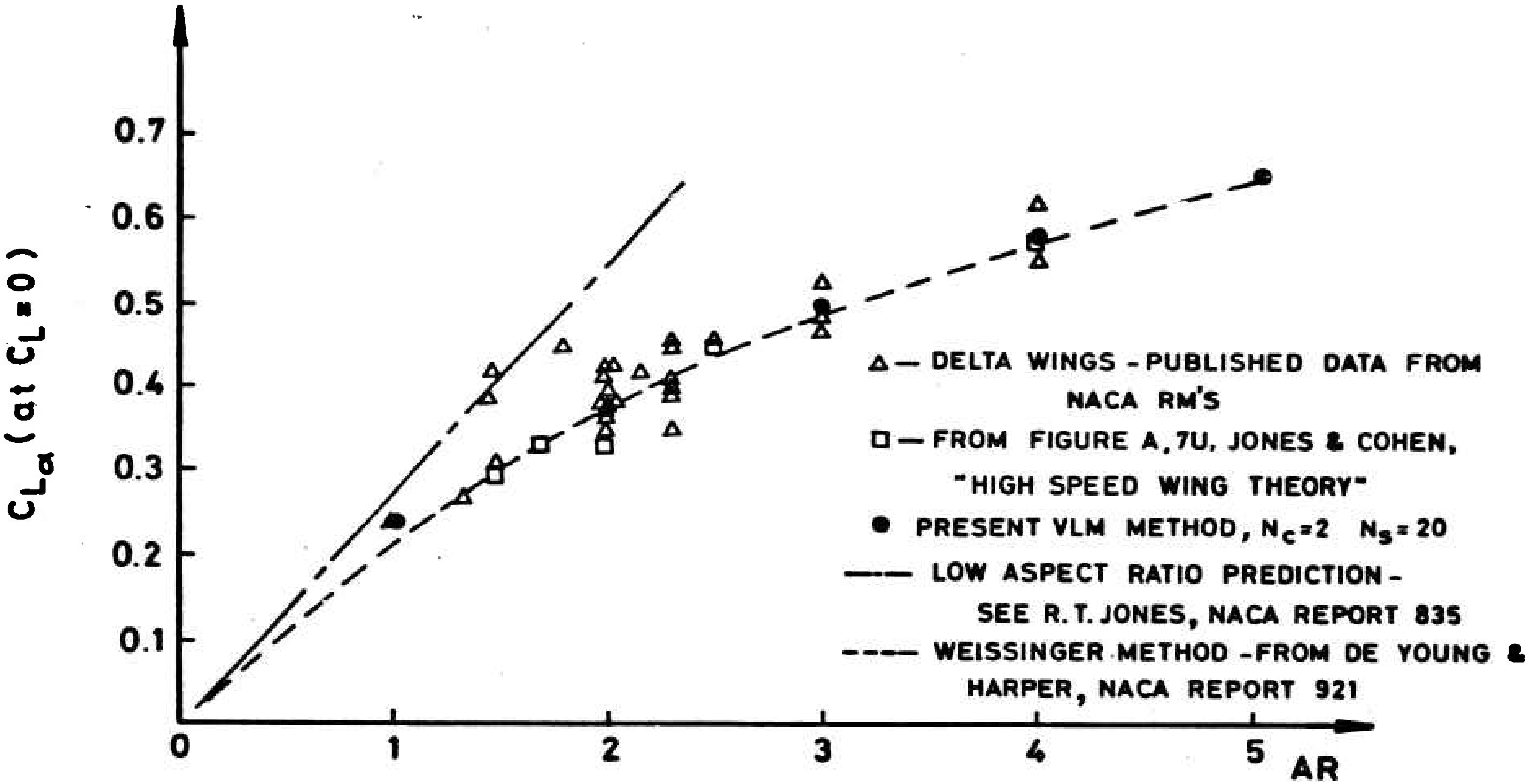

FIGURE 14 - CALCULATED VALUES OF C_{L_α} FOR DELTA WINGS BY THE VLM PROGRAM COMPARED WITH EXPERIMENTAL RESULTS OF REF. (23).

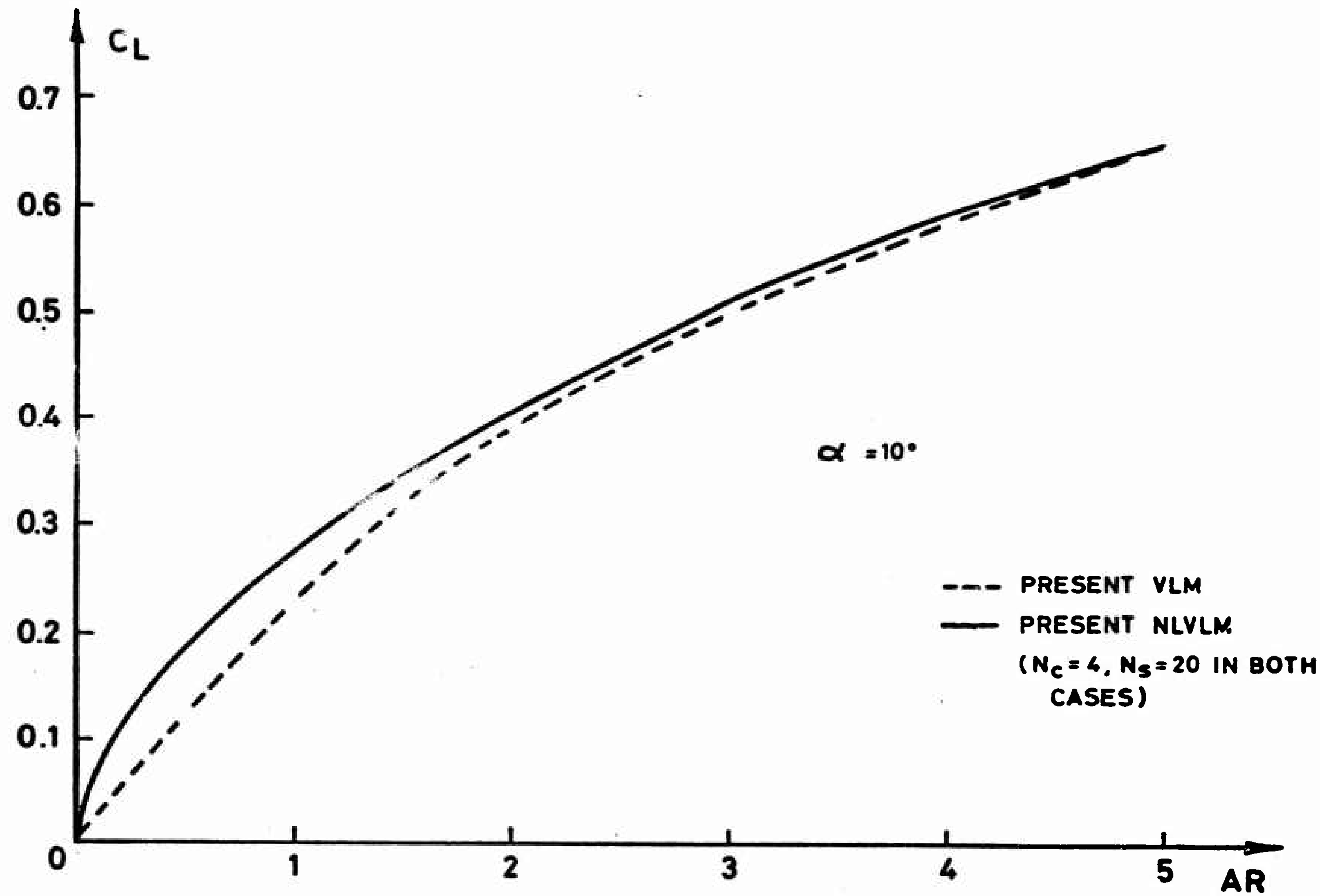

FIGURE 15 - COMPARISON OF THE LINEAR LIFT COEFFICIENT CALCULATED BY THE VLM PROGRAM AND THE NONLINEAR LIFT COEFFICIENT CALCULATED BY THE NLVLM PROGRAM FOR DELTA WINGS AT $\alpha = 10°$

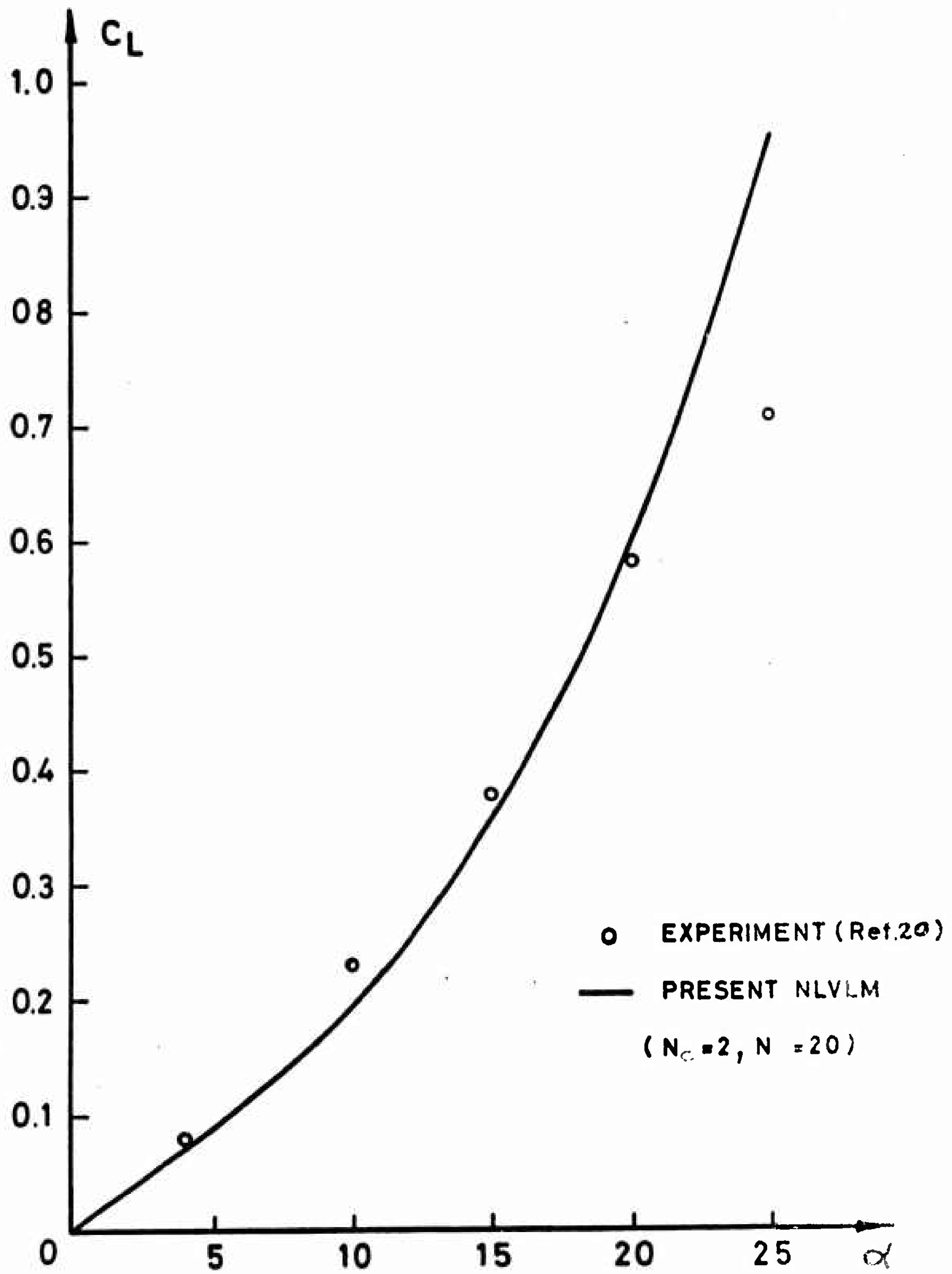

FIGURE 16a - THE LIFT COEFFICIENT OF A DELTA WING OF AR = [illegible]

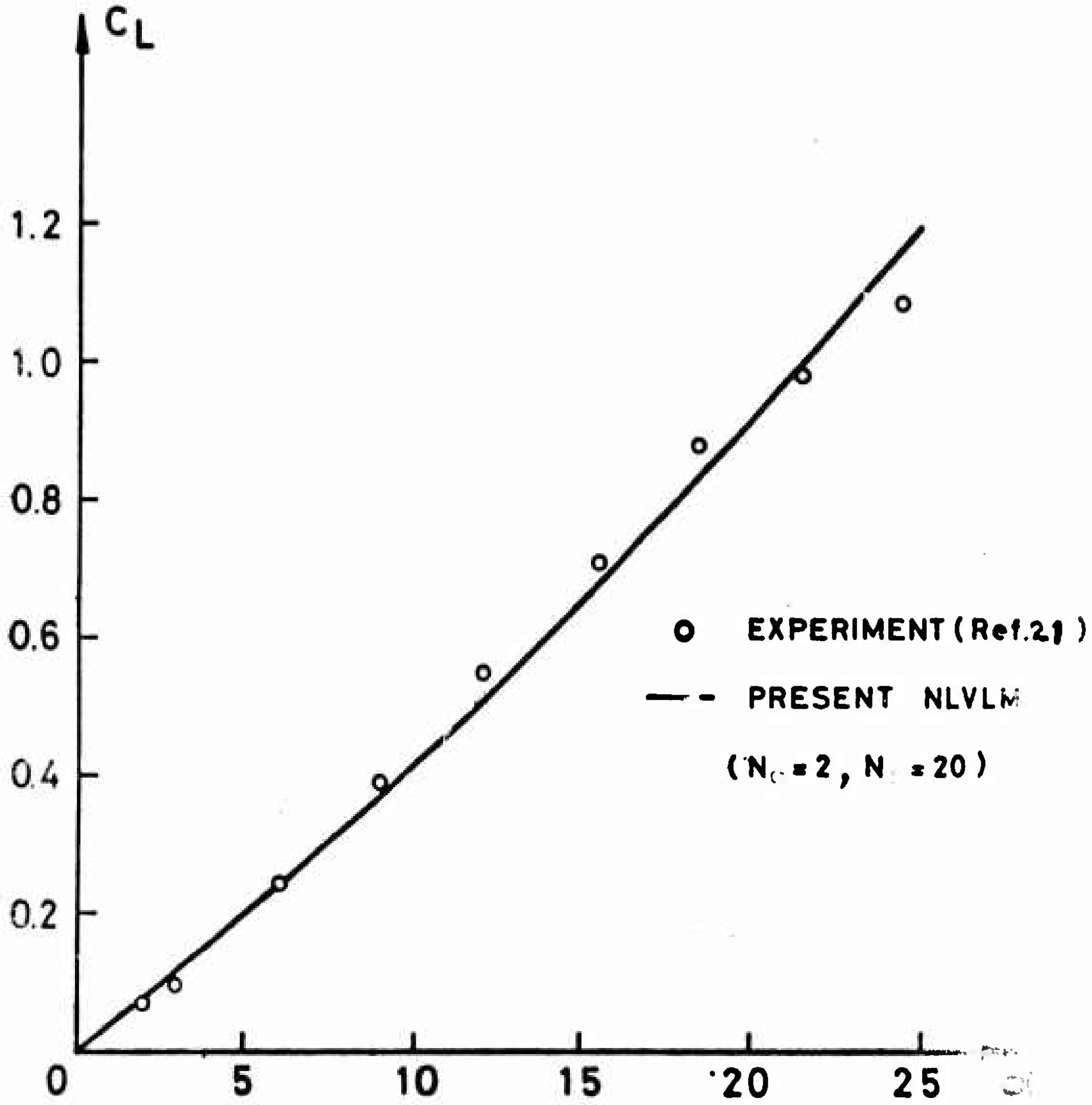

FIGURE [illegible]b - THE LIFT COEFFICIENT OF A DELTA WING OF AR = 2.[illegible]

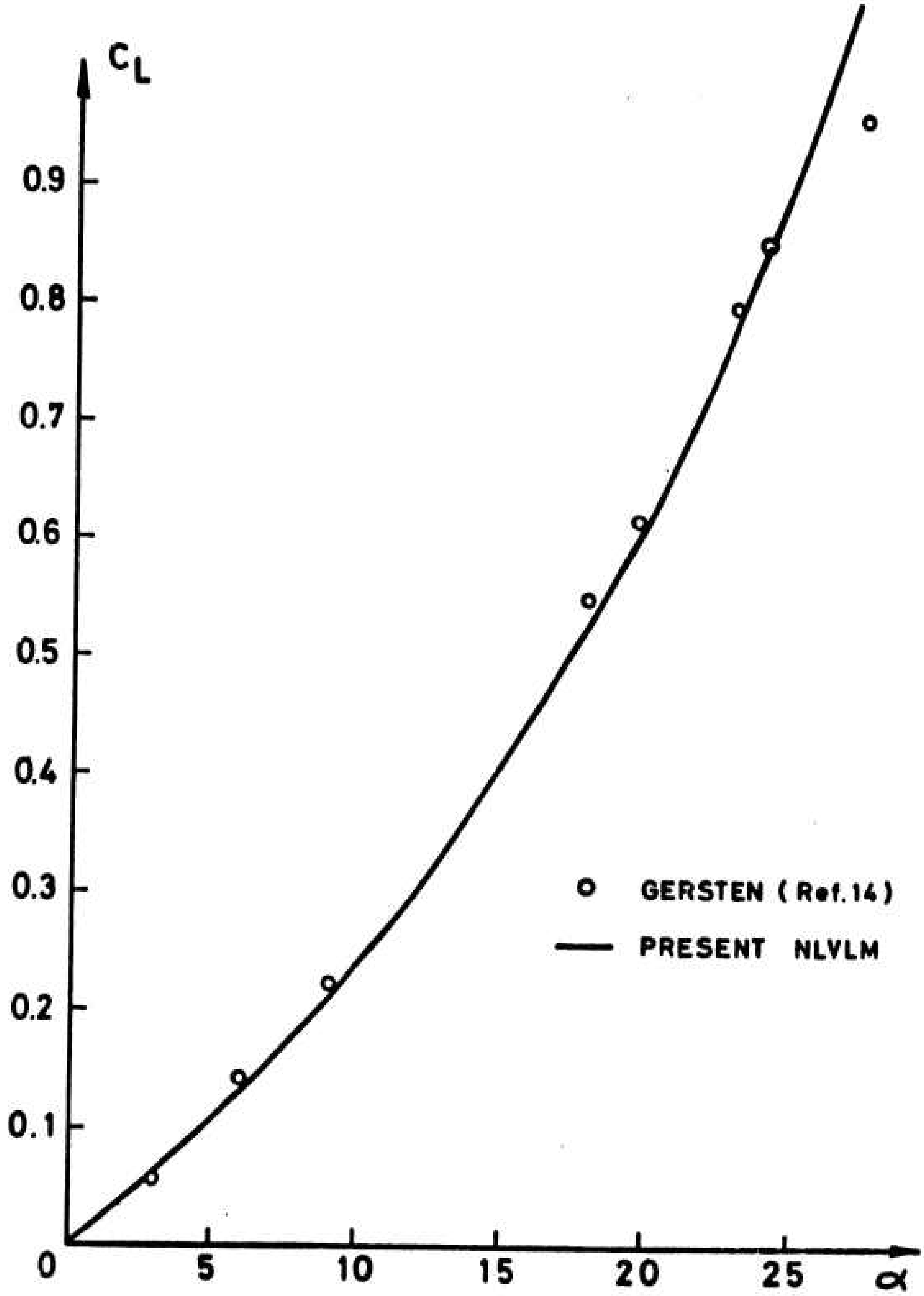

FIGURE 17 - THE LIFT COEFFICIENT OF A CROPPED DELTA WING OF AR = 0.78.

VLM PROGRAM

RECTANGULAR WING AR = 3

J = 10 □ - 2D

N = 20 Δ - 3D

M = 16

M = 32

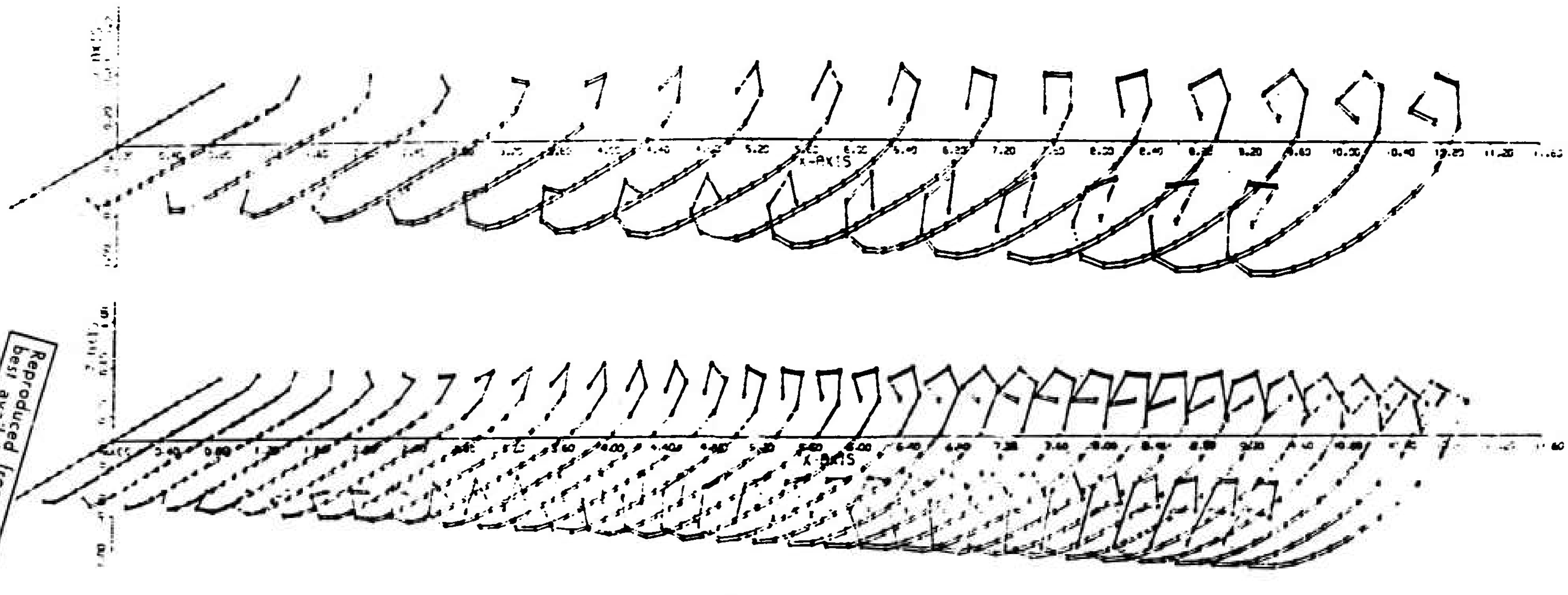

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FIGURE 18 - THE ROLLED-UP WAKE SHAPES CALCULATED BY THE VLM PROCEDURE WITH STEP SIZES M = 16 AND M = 32. (Figure Axes are Given by Half Span Units).

MVLM PROGRAM
Rectangular Wing AR = 3
$\alpha = 10^{o}$
N = 20

M=16

M = 32

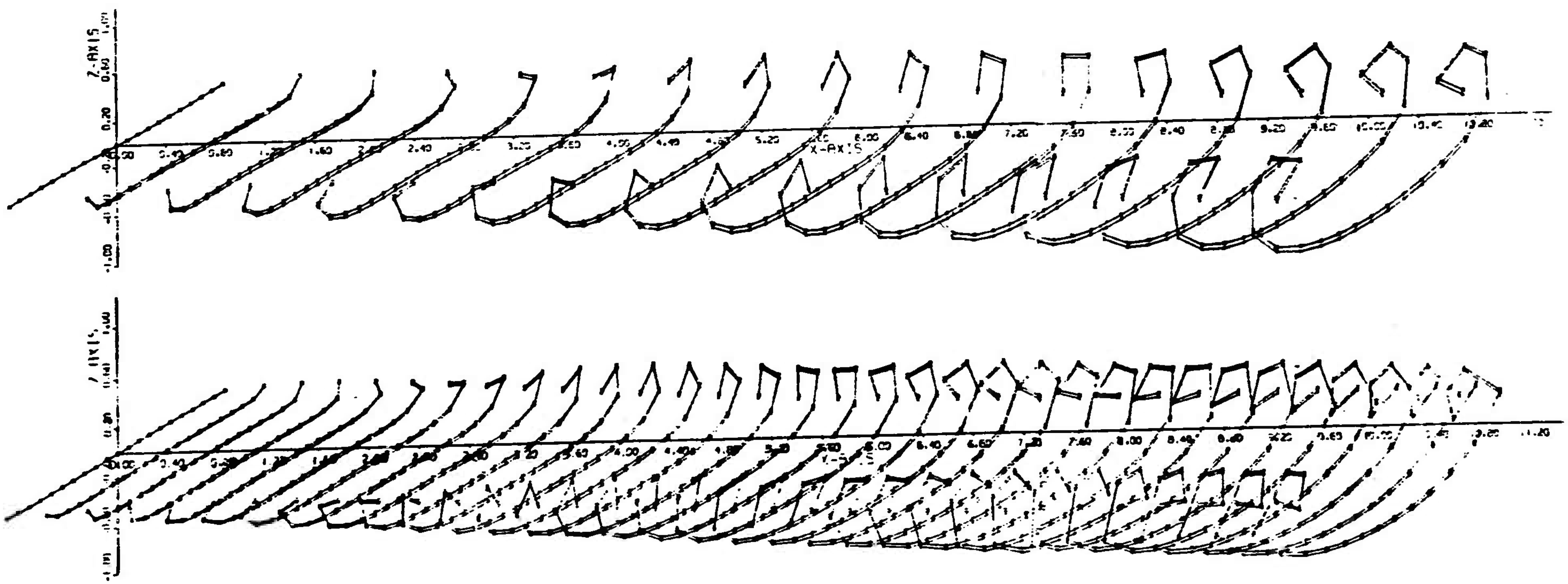

FIGURE 19 - THE ROLLED-UP WAKE SHAPES CALCULATED BY THE MVLM PROCEDURE WITH STEP SIZES M = 16 AND M = 32. (Figure Axes are given in Half Span Units).

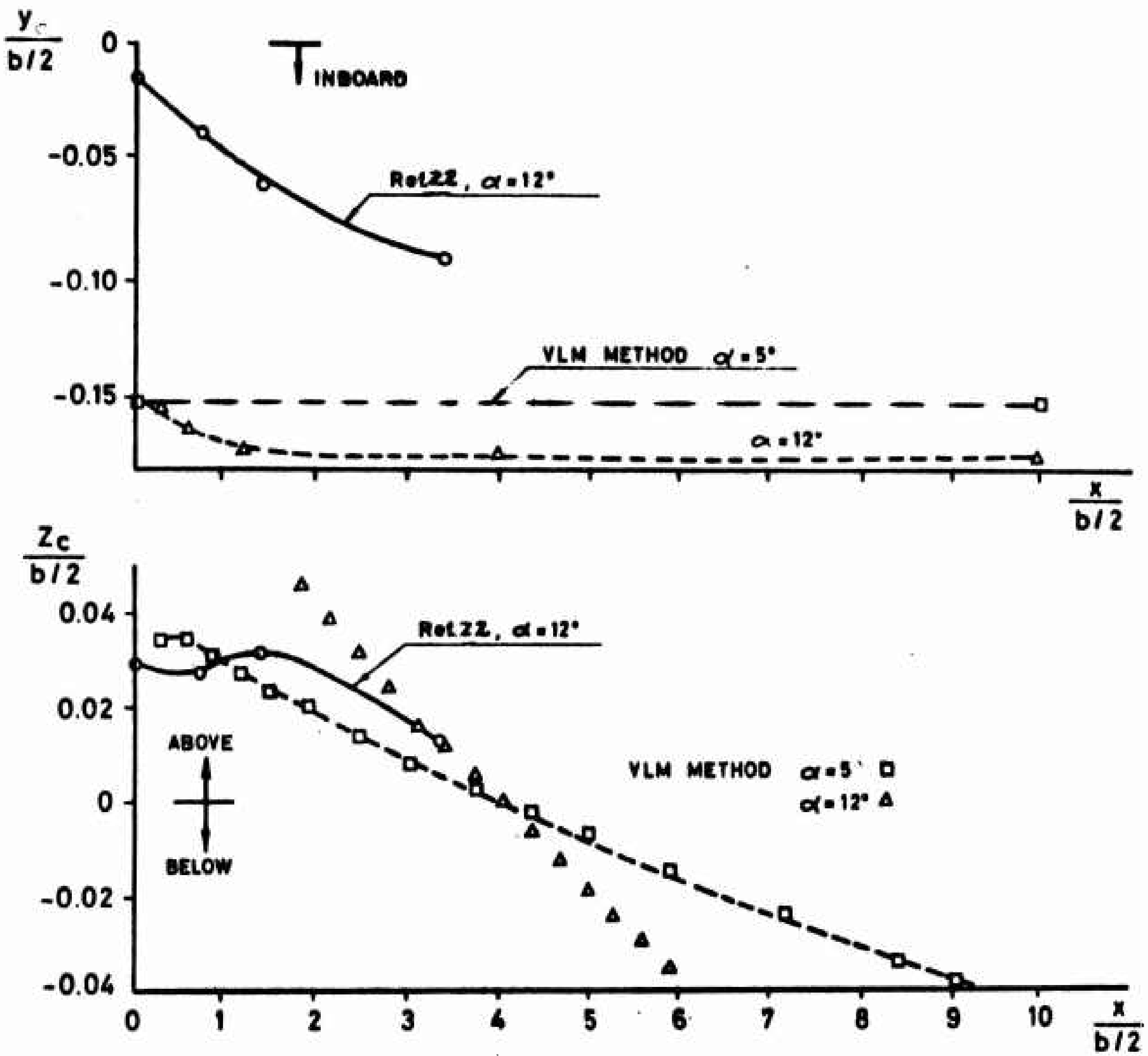

FIG.20 - SPANWISE AND NORMAL LOCATION OF VORTEX CENTER FOR RECTANGULAR WING - AR = 5.33

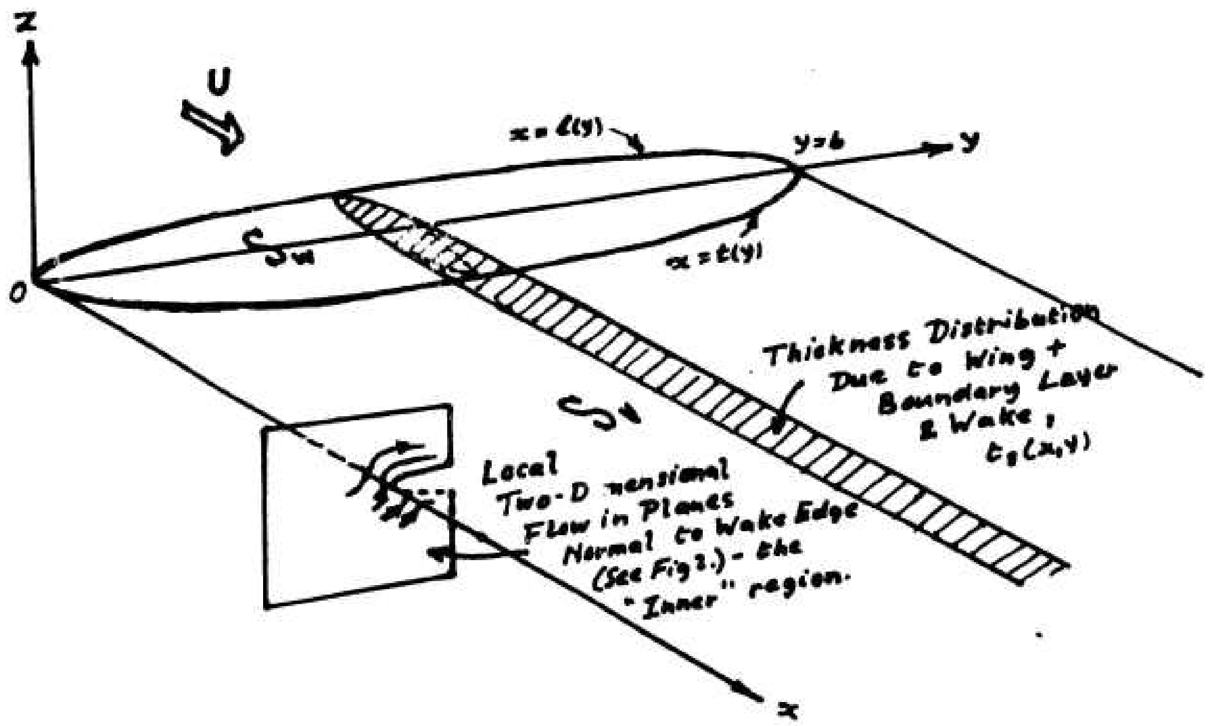

FIGURE 21.

Equation of Parabola:

$$Z = \pm\sqrt{2\lambda Y}$$

Focus is at $(\frac{1}{2}\lambda, 0)$.

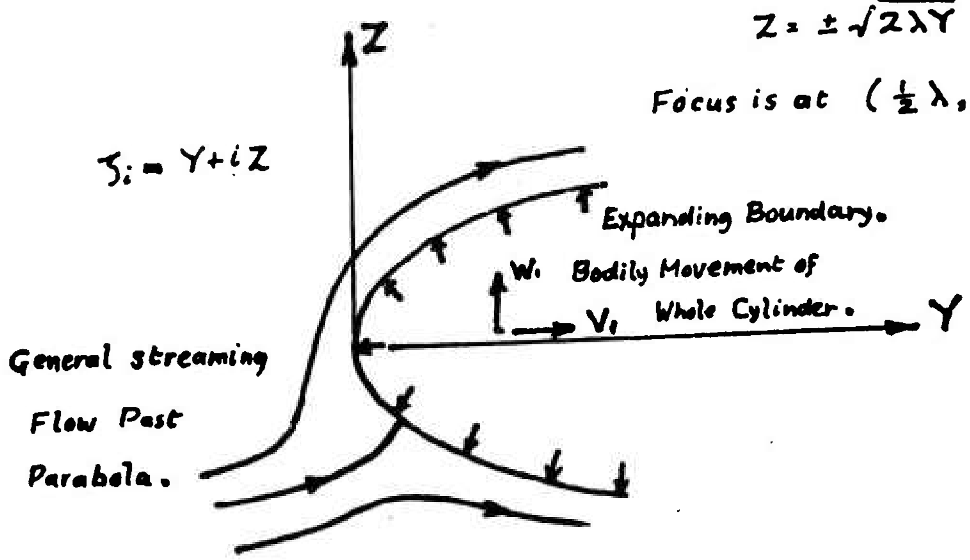

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FIGURE 22.